Election

Learning Objectives

- Solving problems which involve logic and calculating and ordering with whole numbers.
- Understanding and looking critically at differrent voting systems.
- Target Age range: 12-14 years

Resources

You will need one copy of the 'Election' resource sheet, one copy of the 'Election' information sheet and a pair of scissors for each student or group of students.

Description of Activity

Ensure that students understand the five voting systems described on the information sheet. You may wish to assign a different voting system to each group of students and ask them to explain their system to the rest of the class. The five voting systems are organised into increasing levels of difficulty on the information sheet.

Students cut out the eleven ballot papers and use them to decide which candidate is elected under each of the voting systems. They will need to consider how to arrange the ballot papers in piles and how to tabulate the results of the election.

The results of the election should stimulate discussion around the relative merits of each voting system. Emphasise that the choice of voting system can influence the outcome of the election (this ballot has been 'rigged' to ensure each candidate is elected under at least one of the systems).

Solutions

- Relative Majority: Cortez is elected.
  Total votes are Ahmed 2, Boateng 3, Cortez 5, Donald 1
- Absolute Majority: Boateng is elected.
  Total votes after 1st round are Ahmed 2, Boateng 3, Cortez 5, Donald 1 (Donald eliminated).
  Total votes after 2nd round are Ahmed 3, Boateng 3, Cortez 5 (Ahmed eliminated on 1st round votes).
  Total votes after 3rd round are Boateng 6, Cortez 5.
- Borda Points with Arithmetic Weighting: Ahmed is elected.
  Total points are Ahmed 30, Boateng 25, Cortez 26, Donald 29
- Borda Points with Geometric Weighting: Cortez is elected.
  Total points are Ahmed 42, Boateng 38, Cortez 46, Donald 39
- Condorcet Pair-Wise Counting: Donald is elected.
  Eliminate Ahmed and Boateng to give total votes for Cortez 5, Donald 6.
  Eliminate Ahmed and Cortez to give total votes for Boateng 5, Donald 6.
  Eliminate Boateng and Cortez to give total votes for Ahmed 5, Donald 6.

Variations

You could carry out your own election with students nominating their own candidates (you may wish to limit these to a maximum of five) and designing their own ballot papers. Alternatively, you could vote on 'the best film', 'favourite ice cream', etc. This may also give rise to discussions regarding tactical voting.

You could use other voting systems not included on the information sheet, e.g. a 'second ballot'. The Election Reform Society web-site (www.electoral-reform.org.uk) contains useful information on a variety of voting systems. The Inter-Parliamentary Union web-site (www.ipu.org) gives general information on parliaments around the world including a description of the electoral system used.

Election

Cut out the 11 ballot papers and decide which candidate would be elected under each of these voting systems:

- Relative Majority
- Absolute Majority (Alternative Vote)
- Borda Points with Arithmetic Weighting
- Borda Points with Geometric Weighting
- Condorcet Pair-Wise Counting

Ballot Paper

Place a 1 in the box next to your first preference, a 2 in the box next to your second preference, and so on.

Candidate	
Ahmed (Progressive Party)	3
Boateng (Radical Alliance)	4
Cortez (Independent)	1
Donald (Peoples' Forum)	2

Ballot Paper

Place a 1 in the box next to your first preference, a 2 in the box next to your second preference, and so on.

Candidate	
Ahmed (Progressive Party)	2
Boateng (Radical Alliance)	1
Cortez (Independent)	4
Donald (Peoples' Forum)	3

Ballot Paper

Place a 1 in the box next to your first preference, a 2 in the box next to your second preference, and so on.

Candidate	
Ahmed (Progressive Party)	1
Boateng (Radical Alliance)	3
Cortez (Independent)	4
Donald (Peoples' Forum)	2

Ballot Paper

Place a 1 in the box next to your first preference, a 2 in the box next to your second preference, and so on.

Candidate	
Ahmed (Progressive Party)	3
Boateng (Radical Alliance)	4
Cortez (Independent)	1
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Ballot Paper

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Candidate	
Ahmed (Progressive Party)	2
Boateng (Radical Alliance)	3
Cortez (Independent)	1
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Ballot Paper

Place a 1 in the box next to your first preference, a 2 in the box next to your second preference, and so on.

Candidate	
Ahmed (Progressive Party)	3
Boateng (Radical Alliance)	1
Cortez (Independent)	4
Donald (Peoples' Forum)	2

Ballot Paper

Place a 1 in the box next to your first preference, a 2 in the box next to your second preference, and so on.

Candidate	
Ahmed (Progressive Party)	1
Boateng (Radical Alliance)	2
Cortez (Independent)	4
Donald (Peoples' Forum)	3

Ballot Paper

Place a 1 in the box next to your first preference, a 2 in the box next to your second preference, and so on.

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Ahmed (Progressive Party)	2
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Ballot Paper

Place a 1 in the box next to your first preference, a 2 in the box next to your second preference, and so on.

Candidate	
Ahmed (Progressive Party)	3
Boateng (Radical Alliance)	4
Cortez (Independent)	1
Donald (Peoples' Forum)	2

Election

Relative Majority

Often called “First past the post”. Electors have a single vote, shown by an ‘X’, for one candidate only.

- The candidate with the most votes is elected.

Note: On the resource sheet, assume that the first preference (1) on the ballot papers is the single vote (X).

Absolute Majority (Alternative Vote)

Electors vote for candidates using preferences (the first choice is shown by a ‘1’, the second choice by a ‘2’, and so on).

- The candidate with the fewest first preferences is eliminated.
- Each vote from the eliminated candidate is transferred to the second preference.
- This process is repeated until one candidate has more than half of all votes.
- If two candidates have the same number of votes, then the votes from the previous round(s) are used to work out who is eliminated.

Borda Points with Arithmetic Weighting

Electors vote for candidates using preferences (the first choice is shown by a ‘1’, the second choice by a ‘2’, and so on).

- Points are awarded to candidates, according to the electors’ preferences, as shown below.
- If there are four candidates:
  - 4 points are awarded for each first preference
  - 3 points are awarded for each second preference
  - 2 points are awarded for each third preference
  - 1 point is awarded for each fourth preference
- The candidate with the largest points total is elected.

Note: For n candidates, there will be n points for each first preference, n-1 points for each second preference and so on down to 1 point for each nth preference.

Borda Points with Geometric Weighting

Electors vote for candidates using preferences (the first choice is shown by a ‘1’, the second choice by a ‘2’, and so on).

- Points are awarded to candidates, according to the electors’ preferences, as shown below.
- If there are four candidates:
  - 8 points are awarded for each first preference
  - 4 points are awarded for each second preference
  - 2 points are awarded for each third preference
  - 1 point is awarded for each fourth preference
- The candidate with the largest points total is elected.

Note: For n candidates, there will be 2^{n-1} points for each first preference, 2^{n-2} points for each second preference and so on down to 1 point for each nth preference ($2^0=1$).

Condorcet Pair-Wise Counting

Electors vote for candidates using preferences (the first choice is shown by a ‘1’, the second choice by a ‘2’, and so on).

- For each pair of candidates, the preferences for those two candidates only are considered.
- If a candidate is preferred to all other candidates, then that candidate is elected.
- If there is no such candidate, then another voting system must be used.

Note: For four candidates, there will be six pairs to consider.

Election (Starter)

Here are three ballot papers from an election for a single candidate to represent a group of voters.

If these were the only three votes cast, which candidate do you think should be elected? Why?

Ballot paper 1

Candidate	Preference
Banda	1
Kirchner	5
McAleese	4
Patil	2
Roussef	3

Ballot paper 2

Candidate	Preference
Banda	5
Kirchner	4
McAleese	3
Patil	2
Roussef	1

Ballot paper 3

Candidate	Preference
Banda	2
Kirchner	4
McAleese	1
Patil	3
Roussef	5

Prompts:

Note: 1 = 1st preference (most favoured candidate), …
5 = 5th preference (least favoured candidate).

- What if you gave 5 points for each 1st preference, 4 for each 2nd preference, 3 for each 3rd preference, and so on down to 1 for each 5th preference?
- What if you gave 16 points for each 1st preference, 8 for each 2nd preference, 4 for each 3rd preference, and so on down to 1 for each 5th preference?
- Is there a fairer way of counting the votes?

Teachers Notes:

- Allow students to discuss first of all which candidate should be elected and why (without yet considering the prompts on Slide 2).
- Encourage students to come up with their own justifications based on fairness.
- Note that no candidate will win outright under the relative majority or absolute majority methods of counting the votes (see information sheet in Election activity).
- The system of allocating points for different preferences was devised by Jean-Charles de Borda (1733-1799) - French mathematician, physicist, political scientist and sailor.
- The most common ways of allocating Borda points are by using either:
  - arithmetic weighting (for 5 candidates, 5 points are allocated to each 1st preference, 4 to each 2nd, 3 to each 3rd, 2 to each 4th, 1 to each 5th);
  - geometric weighting (for 5 candidates, 16 points are allocated to each 1st preference, 8 to each 2nd, 4 to each 3rd, 2 to each 4th, 1 to each 5th).
- Encourage students to devise their own sequence for allocating points.
- Encourage students to discuss which method is fairest.
- Note that using Borda Points with arithmetic weighting is more likely to lead to a consensus candidate being elected.
- Note that the names selected on the ballot papers are those of current or former female Presidents from Malawi, Argentina, Ireland, India, Brazil.

TMSJ Research Group - Notes on proposed activity for cycle 3 (May-July 2014)

Election activity

Rationale for the activity:

- Sixth formers seem cynical about voting - they don't trust politicians.
- People don't understand AV system, hence were apathetic towards the recent referendum.

Aims of the activity:

- Try to convince students that they should vote (or at least, if they choose not to vote, it is from a considered position).
- Develop understanding of different voting systems (*including the maths underlying them*).
- Getting them to vote on something they feel strongly about potentially gets them to realise how relevant and powerful maths can be.

Structure of activity:

- Use the Election activity from HRinC to show how different candidates can win if different systems are used.
- Extend to voting on something that is relevant to students' real lives/interests, e.g. deciding the class's 'favourite film' (see teachers' notes from Election activity).
- Choosing films avoids the need to vote for individual students which might be problematic.
- Take nominations and then discuss which voting system to use, i.e. which is fairest and why?
- Might need to create ballot papers once nominations have been made for students to vote.
- *Carry out the election and reflect on the outcome. Was it fair? Why?*
- Discuss tactical voting, e.g. you can split the vote under some methods by nominating a very similar film to the one you don't want to win (nominate Hobbit 2 to stop Hobbit 1 winning).

Extensions:

- Relate to which systems we use in UK elections? *How fair is our 'first-past-the-post' system?*
- *What is the smallest percentage of the vote a party could win and still win general election?*
- Look at AV voting (could use 'Cats versus dogs' video).
- Look at data on turnout in elections, e.g. South Africa ('Born Free's in the news recently), East Timor.
- *Should voting be compulsory (as in Australia)?*
- Picture of first free South African election in 1994: